

6.5 Trapezoids and Kites

- Goals**
- Use properties of trapezoids.
 - Use properties of kites.

VOCABULARY

Trapezoid

Bases of a trapezoid

Base angles of a trapezoid

Legs of a trapezoid

Isosceles trapezoid

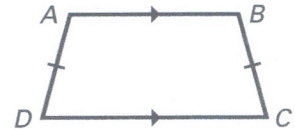
Midsegment of a trapezoid

Kite

THEOREM 6.14

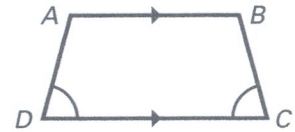
If a trapezoid is isosceles, then each pair of base angles is _____.

$$\angle A \cong \angle _, \angle _ \cong \angle D$$

**THEOREM 6.15**

If a trapezoid has a pair of congruent _____, then it is an isosceles trapezoid.

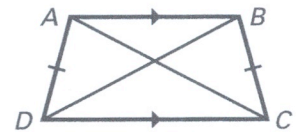
$ABCD$ is an isosceles trapezoid.

**THEOREM 6.16**

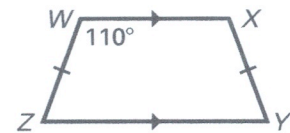
A trapezoid is isosceles if and only if its diagonals are _____.

$ABCD$ is isosceles if and only

if $_ \cong _$.

**Example 1** *Using Properties of Isosceles Trapezoids*

$WXYZ$ is an isosceles trapezoid.
Find $m\angle X$, $m\angle Y$, and $m\angle Z$.

**Solution**

- $WXYZ$ is an isosceles trapezoid,
so $m\angle X = m\angle _ = _^\circ$.
- $\angle W$ and $\angle Z$ are consecutive interior angles formed by parallel lines, so they are _____.
 $m\angle W + m\angle Z = _^\circ$ **Consecutive Interior Angles Theorem**
 $_^\circ + m\angle Z = _^\circ$ **Substitute for $m\angle W$.**
 $m\angle Z = _^\circ$ **Subtract $_^\circ$ from each side.**
- $WXYZ$ is an isosceles trapezoid, so $m\angle Y = m\angle _ = _^\circ$.

Example 2 Using Properties of Trapezoids

Show that $HIJK$ is a trapezoid.

Compare the slopes of opposite sides.

Slope of \overline{HK} = _____ = _____ = _____

Slope of \overline{IJ} = _____ = _____ = _____

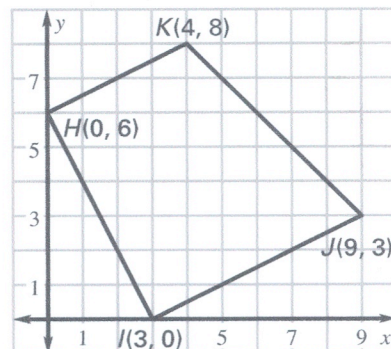
The slopes of \overline{HK} and \overline{IJ} are equal,
so $\overline{HK} \parallel \overline{IJ}$.

Slope of \overline{HI} = _____ = _____ = _____

Slope of \overline{JK} = _____ = _____ = _____

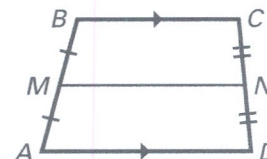
The slopes of \overline{HI} and \overline{JK} are not equal, so \overline{HI} is not parallel to \overline{JK} .

Answer Because $\overline{HK} \parallel \overline{IJ}$ and \overline{HI} is not parallel to \overline{JK} , $HIJK$ is a _____.



THEOREM 6.17: MIDSEGMENT THEOREM FOR TRAPEZOIDS

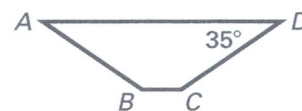
The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.



$\overline{MN} \parallel$ _____, $\overline{MN} \parallel$ _____, $MN =$ _____

✓ **Checkpoint** Complete the following exercise.

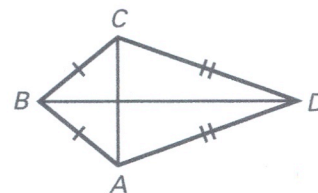
1. $ABCD$ is an isosceles trapezoid.
Find $m\angle A$, $m\angle B$, and $m\angle C$.



THEOREM 6.18

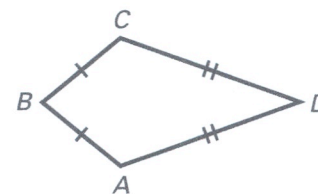
If a quadrilateral is a kite, then its diagonals are _____.

_____ \perp _____

**THEOREM 6.19**

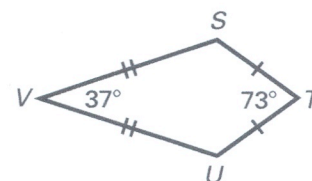
If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

$\angle A$ _____ $\angle C$, $\angle B$ _____ $\angle D$

**Example 3** *Angles of a Kite*

Find $m\angle S$ and $m\angle U$.

$STUV$ is a kite, so $\angle S \cong \angle$ _____ and $m\angle S = m\angle$ _____.



$$\text{_____} (m\angle S) + m\angle \text{_____} + m\angle \text{_____} = \text{_____}^\circ$$

Sum of measures of int. \angle s of quad. is 360° .

$$\text{_____} (m\angle S) + \text{_____}^\circ + \text{_____}^\circ = \text{_____}^\circ$$

Substitute.

$$\text{_____} (m\angle S) = \text{_____}^\circ$$

Simplify.

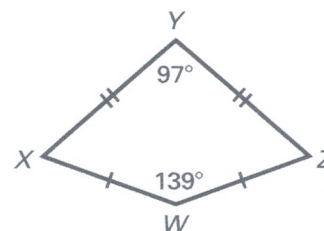
$$m\angle S = \text{_____}^\circ$$

Divide each side by _____.

Answer So, $m\angle S = m\angle$ _____ = _____ $^\circ$.

✓ **Checkpoint** Complete the following exercise.

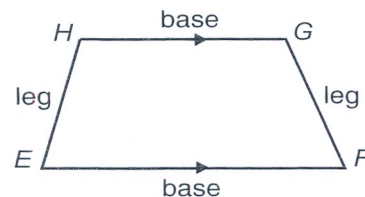
2. Find $m\angle X$ and $m\angle Z$.



Study Guide

Trapezoids

A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**, and the nonparallel sides are called **legs**. In trapezoid $EFGH$, $\angle E$ and $\angle F$ are called **base angles**. $\angle H$ and $\angle G$ form the other pair of base angles.



A trapezoid is an **isosceles trapezoid** if its legs are congruent.

The **median** of a trapezoid is the segment that joins the midpoints of the legs.

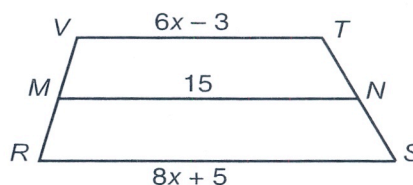


The following theorems all concern trapezoids.

- Both pairs of base angles of an isosceles trapezoid are congruent.
- The diagonals of an isosceles trapezoid are congruent.
- The median of a trapezoid is parallel to the bases, and its measure is one-half the sum of the measures of the bases.

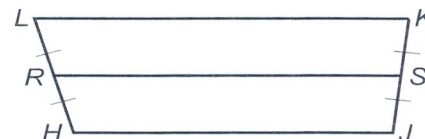
Example: Given trapezoid $RSTV$ with median \overline{MN} , find the value of x .

$$\begin{aligned} MN &= \frac{1}{2}(VT + RS) \\ 15 &= \frac{1}{2}(6x - 3 + 8x + 5) \\ 15 &= \frac{1}{2}(14x + 2) \\ 15 &= 7x + 1 \\ 14 &= 7x \\ 2 &= x \end{aligned}$$



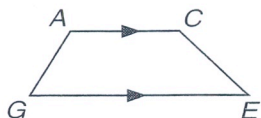
$HJKL$ is an isosceles trapezoid with bases \overline{HJ} and \overline{LK} , and median \overline{RS} . Use the given information to solve each problem.

1. If $LK = 30$ and $HJ = 42$, find RS .
2. If $RS = 17$ and $HJ = 14$, find LK .
3. If $RS = x + 5$ and $HJ + LK = 4x + 6$, find RS .
4. If $m\angle LRS = 66$, find $m\angle KSR$.

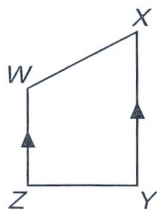


Skills Practice**Trapezoids***For each trapezoid, name the bases, the legs, and the base angles.*

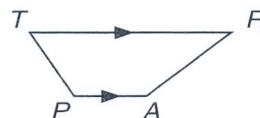
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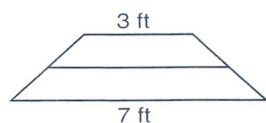
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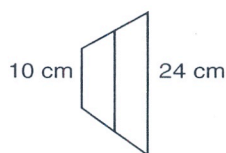
3.

*Find the length of the median in each trapezoid.*

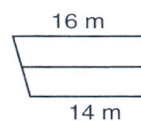
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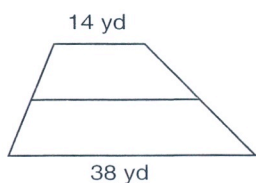
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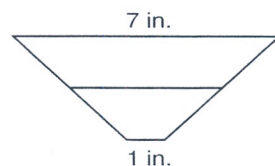
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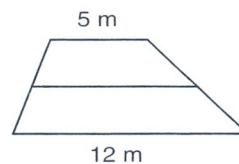
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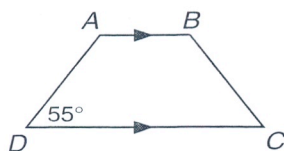
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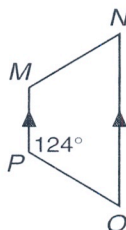
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*Find the missing angle measures in each isosceles trapezoid.*

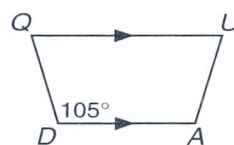
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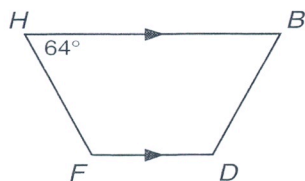
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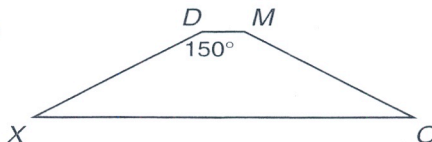
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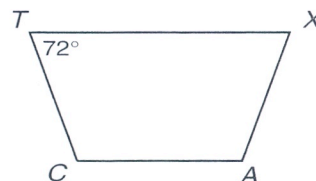
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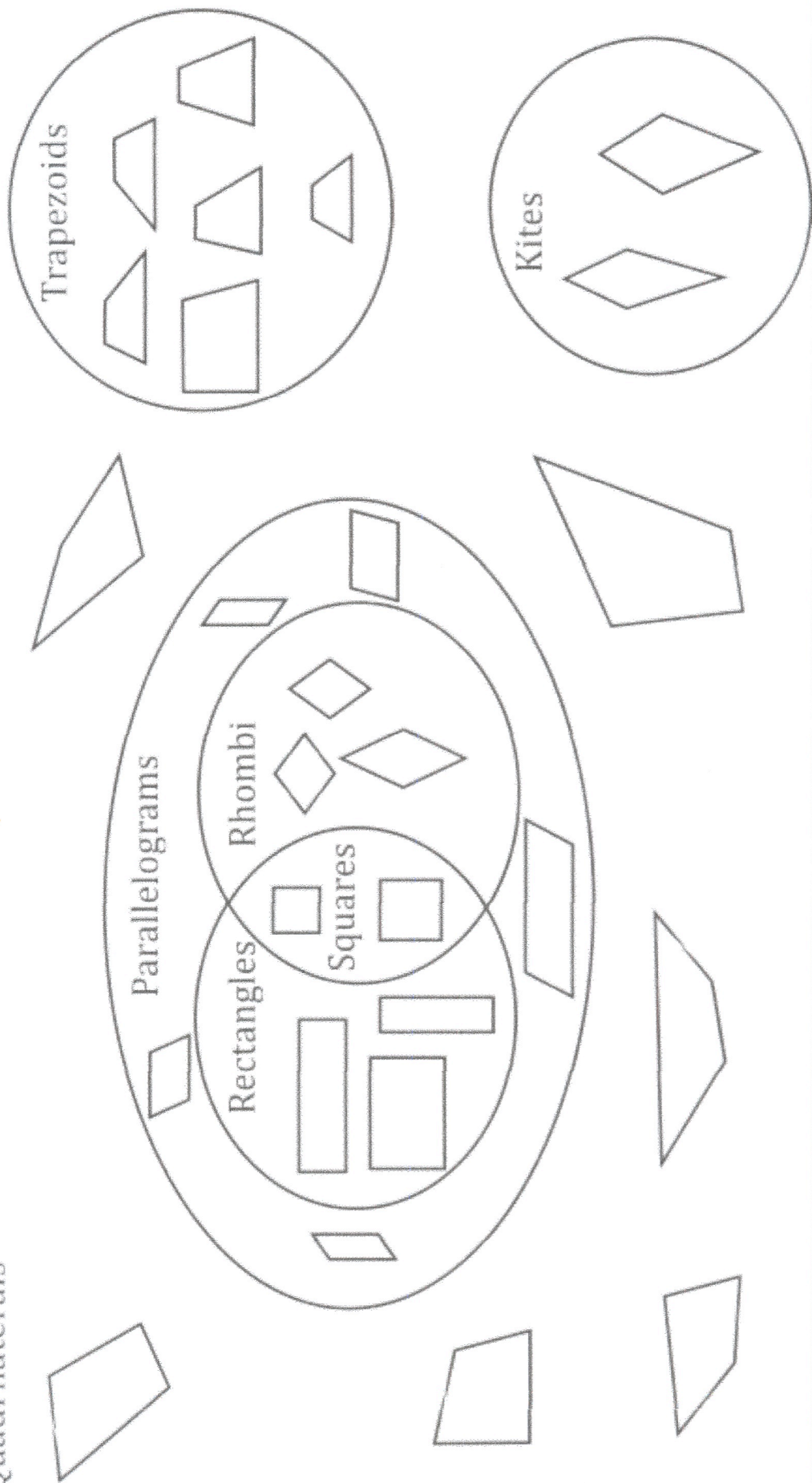
14.



15.



Quadrilaterals



7. A rhombus is always a square.

9. All parallelograms are also squares.

11. A kite is always a rhombus.

13. A rectangle is both a parallelogram and a square.

15. All parallelograms are rectangles.

17. If a figure is a trapezoid then it is a parallelogram.

8. All squares are parallelograms.

10. If a figure is a quadrilateral, then it is a parallelogram.

12. All quadrilaterals are kites.

14. A square is both a parallelogram and a rectangle.

16. A parallelogram could be square.

18. A square is rectangle, a rhombus, a parallelogram and a trapezoid.

#1.	#2.	#3.	#4.	#5.	#6.	#7.	#8.	#9.	#10	#11	#12.	#13	#14	#15	#16	#17	#18
OT	OT	OT	OT	OT	OT	OT	OT	OT	OT	OT	OT	OT	OT	OT	OT	OT	OT
OF	OF	OF	OF	OF	OF	OF	OF	OF	OF	OF	OF	OF	OF	OF	OF	OF	OF

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For these choose the answer that is the most specific name for the quadrilateral. (Draw a picture first!)

31. In quadrilateral ABCD $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$.

Quadrilateral ABCD is a...

- A. Rectangle
- B. Parallelogram
- C. Square
- D. Trapezoid

33. In quadrilateral IJKL \overline{IK} and \overline{JL} are perpendicular and bisect each other. Quadrilateral IJKL is a...

- A. Rhombus
- B. Parallelogram
- C. Rectangle
- D. Square

35. In quadrilateral QRST $\overline{QS} \cong \overline{RT}$. Quadrilateral QRST is a...

- A. Rhombus
- B. Quadrilateral
- C. Rectangle
- D. Square

37. In quadrilateral ABCD $\overline{AB} \cong \overline{CD} \cong \overline{BC} \cong \overline{DA}$, and all angles are 90° . Quadrilateral ABCD is a...

- A. Rectangle
- B. Square
- C. Parallelogram
- D. Rhombus

39. Quadrilateral IJKL is equilateral. Quadrilateral IJKL is a...

- A. Rhombus
- B. Trapezoid
- C. Rectangle
- D. Square

32. In quadrilateral EFGH EG and FH bisect each other. Quadrilateral EFGH is a...

- A. Quadrilateral
- B. Square
- C. Parallelogram
- D. Rectangle

34. In quadrilateral MNOP $\overline{MN} \parallel \overline{OP}$. Quadrilateral MNOP is a...

- A. Square
- B. Parallelogram
- C. Rectangle
- D. Trapezoid

36. In quadrilateral UVWX $\overline{UW} \cong \overline{VX}$, \overline{UW} bisects \overline{VX} , and $\overline{UW} \perp \overline{VX}$. Quadrilateral UVWX is a...

- A. Rhombus
- B. Quadrilateral
- C. Rectangle
- D. Square

38. In quadrilateral EFGH $\angle E \cong \angle G$, and $\angle F \cong \angle H$. Quadrilateral EFGH is a...

- A. Rhombus
- B. Quadrilateral
- C. Parallelogram
- D. Square

40. In quadrilateral MNOP $\angle P$ and $\angle M$ are supplementary. Quadrilateral MNOP is a...

- A. Rhombus
- B. Trapezoid
- C. Rectangle
- D. Square

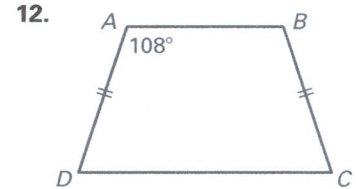
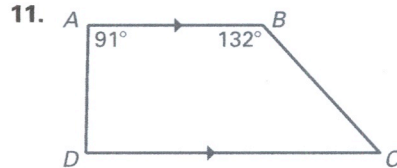
Bubble the correct answer choice from each item above.

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
<input type="radio"/> A.	<input type="radio"/> A.	<input type="radio"/> A.	<input type="radio"/> A.	<input type="radio"/> A.	<input type="radio"/> A.	<input type="radio"/> A.	<input type="radio"/> A.	<input type="radio"/> A.	<input type="radio"/> A.
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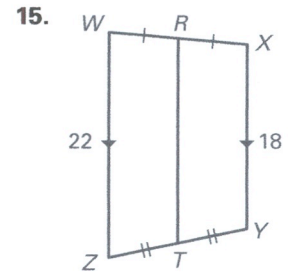
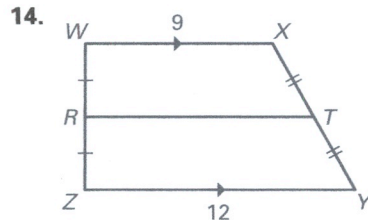
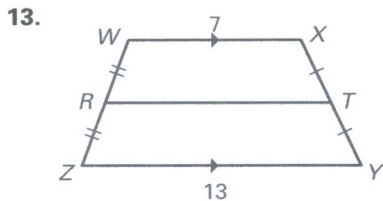
8. The base angles of an isosceles trapezoid are ? congruent.

9. The legs of a trapezoid are ? congruent.

Find the angle measures of $ABCD$.



Find the length of the midsegment \overline{RT} .



Find the length of the sides to the nearest hundredth or the measure of the angles in kite $KITE$.

